Applications of Modern Physics: Loop Quantum Gravity

P. M. Jackson*

Department of Physics, Auburn University, Auburn, AL, USA

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Abstract

As currently understood, two of the most successful theories in physics - quantum mechanics (QM) and Einstein's theory of general relativity (GR) - are incompatible. Many theories are being developed which seek to rectify their incompatibilities and form a single unified theory of Quantum Gravity. Such theories aim to describe the nature of spacetime itself at the quantum level where probabilities, uncertainties, and quantum fluctuations can no longer be ignored. Two of the most well know theories of quantum gravity include Loop Quantum Gravity (LQG) and String Theory. The two take vastly different approaches to solving the same problem, though this paper will only explore Loop Quantum Gravity. This paper seeks to provide the reader with an basic understanding of the core elements of how Loop Quantum Gravity aims to merge quantum mechanics and general relativity into a unified theory of quantum gravity.

I. BACKGROUND

A theory of quantum gravity, such as LQG, does not emerge as a result of observation or experiment, but rather as a necessity to fully understand the consequences and mingling of other established theories. Many physical theories have been successfully developed to reconcile seemingly incompatible differences: Newton's theory of universal gravitation was a result of combining Galileo's Earth-based kinematics with Kepler's laws of orbital motion [1]; electromagnetism from Maxwell's unification of electricity, magnetism, and light [1]; special relativity from Einstein's resolution of electrodynamics and classical mechanics; and similarly, Einstein's general relativity from Newton's gravity and his own special relativity. In the same way, a theory of quantum gravity seeks to unify the smooth and continuous nature of general relativity's spacetime with the discrete and uncertain nature of quantum field theory (QFT).

* pmj0008@auburn.edu

A. Why do we need a quantum theory of gravity?

At it's most basic level, the need for a quantum theory of gravity arises from conceptual inconsistencies between GR and QFT. With GR, Einstein discovered that spacetime and the gravitational field were one in the same - meaning that spacetime is the physical manifestation of gravity. It is also understood from GR that this spacetime on which all things exist, is not a static, unmoving background, but a malleable and continuous fabric that can stretch, squeeze, and oscillate. QFT says that all fields exhibit quantum properties such as having quantities which are restricted to discrete values, having a limit placed on how accurately a physical quantity can be measured, and being subject to probability [2]. Because spacetime is a manifestation of the gravitational field, QFT implies that there must be some minimum quantum of space and a minimum quantum of time which exists [3]. These ideas of a continuous spacetime as described by GR and quantized spacetime by QM/QFT are in direct conflict with one another. However, both theories have, independently, been confirmed over and over again by experiment, so both must be carefully examined and considered to create a bridge between them. There are many other, more technical reasons for incompatibilities between the two theories which are not mentioned here - see section I.1.1 of [4].

B. The first hint of quantized space

The first evidence that space may be quantized arose in the mid 1930's with the surprisingly straightforward application of Heisenberg's uncertainty principle to GR. Heisenberg's uncertainty principle is given by:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{1}$$

where Δx is the uncertainty in position measurement, Δp is the uncertainty in momentum, and $\hbar$ is reduced Planck's constant $\frac{h}{2\pi}$. Now say we want to extract information from an area with precision L, we can rearrange Eq. 1 to

$$\Delta p \geq \frac{\hbar}{2L} \tag{2}$$

Measuring this small area L will require some amount of energy E. From the mass-energy and energy-momentum relations, $M = \frac{E}{c^2}$ and $E \sim cp$, where c is the speed of light. General Relativity says that mass (and energy) curve spacetime and that if enough energy is compacted into a small enough region, a black hole with Schwarzchild Radius $R = \frac{2GM}{c^2}$ (where G is Newton's constant of gravitation) will form. This implies that the most localized a measurement can be is when $L = R$, any more localized and the energy density will create a mini black hole, resulting in the loss of localization. Substituting these relations into eq. 2:

$$L = R = \frac{2GM}{c^2} = \frac{2GE}{c^4} = \frac{2Gp}{c^3} = \frac{G\hbar}{Lc^3} \tag{3}$$

Simply solving for L, we find:

$$l_{planck} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35}m \tag{4}$$

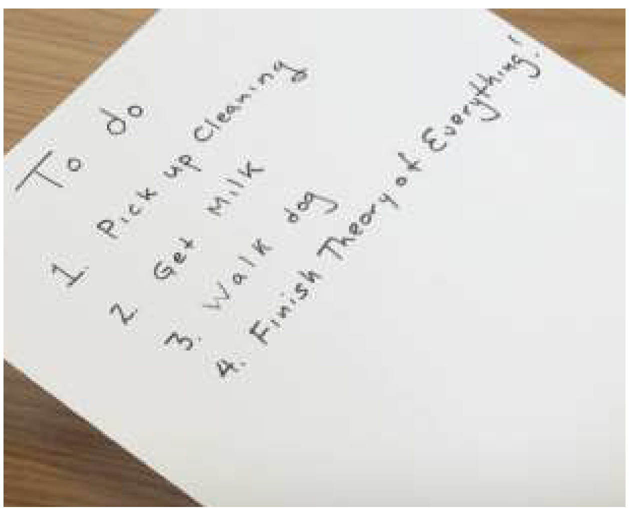
which constitutes the smallest region of space that can be localized [2]. This is the Planck length, which was initially developed by Max Planck in a different and unrelated way to quantum gravity. The existence of a minimum allowed length value within space has important consequences. First, it implies that space quite literally does not exist at any smaller scales - the idea of distance no longer makes sense below the Planck length [2, 5, 6]. Thus, at the Planck-scale, space begins to exhibit a more quantum behavior - becoming discrete and no longer a smooth manifold, which GR assumes, and probabilistic quantum fluctuations begin to dominate [2, 7]. Lastly, it implies the existence of discrete spectra of area and volume - in a similar way to the Planck length, no such area can exist below the Planck area and no volume can exist below the Planck volume (these ideas will be expanded upon later in the paper) [5, 7].

C. Background Independence

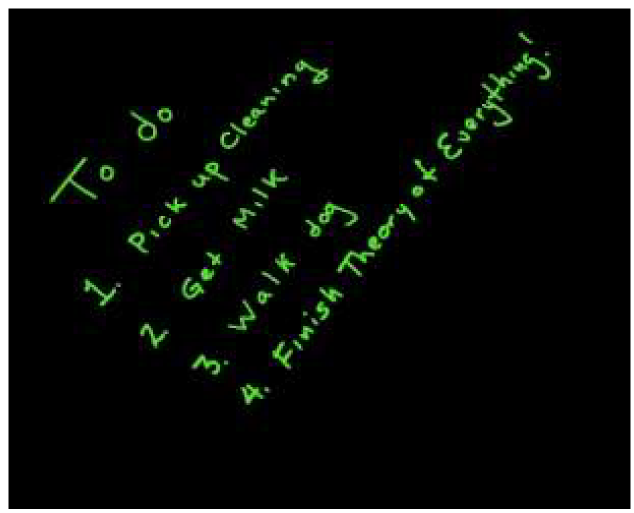
One of the defining aspects of LQG is that it is a background-independent theory. This is derived from the fact that GR is also a background-independent theory and that LQG aims to conserve this property [5]. Many other theories, such as Newtonian mechanics, QM and QFT, and string theory are background dependent.

Background independence (BI) vs. background dependence (BD), while vital to all physical theories, is a debate that tends to reach slightly into the realm of philosophy and metaphysics [8]. First, I will describe BD as it is the more natural of the two to think about. BD assumes that all entities gain properties which are defined with respect to a single, universal, unchanging entity known as the background. Newtonian mechanics, for example, assumes that there exists a static three-dimensional space on which all things exist and an absolute time from which all things evolve. The properties of a ball shot from a cannon such as its position, acceleration, and momentum are all defined with respect to some unchanging background space and time.

On the other hand, properties within a BI theory arise as a result of ones relationship with other entities and how these relationships change with time - a.k.a. there exists no static, universal background which to derive properties. A good way to think about BI is to imagine you have a piece of paper and you write a grocery list on it. This paper with writing can be though of a representation of BD, where the paper itself is the background from which the writing derives its properties - shape, size, letter and word spacing. Now, imagine the paper disappears, but the writing remains - this would be a BI case where there exists no background and the words and letters in the list must derive their properties from the other words and letters around them. [5, 8]



(a) The paper on which the list is written can be thought of as the background, or coordinate system, which the writing derives its properties such as shape, size, and spacing. This can be though of as a background dependent system. Image taken from [9]



(b) In the background independent system, the background coordinate system (paper) no longer exists and the letters must now rely on its relationship to the other letters in order to derive its properties. Image taken from [9]

Figure 1: Example of a background dependent system versus a background independent system

The importance of maintaining background independence, as LQG does, is that background independence is a core property of space itself, as discovered by Einstein in his theory of general relativity. This property must be maintained to ensure that the resulting theory is true to the nature of reality as it is currently understood. For example, in the 1913 Niels Bohr introduced his model of atomic hydrogen. This model, unlike those previous to him, incorporated Max Planck's idea of quantization. Bohr described atomic hydrogen with discrete energy levels such that the orbiting electron could only exist at these discrete levels and not in between. Though this model only describes the hydrogen atom in full, the introduction of quantized energy levels became a vital property of the atomic structure. All currently accepted theories of atomic structure, in some way, maintain the property of quantized energy levels because it accurately describes experimental results and thus, is interpreted to accurately describe nature. This implies that an atomic model lacking quantized energy levels is *not* accurate to describing the nature of matter. That is not to say it is impossible to produce a model that does not include quantization which is also capable of accurately predicting experiment, but as it is currently understood, quantization *is* a core property of the atomic structure. Background independence is to general relativity as quantization is to atomic theory. Background independence is accepted to be a basic property of space, so any future theory describing the nature of space itself, must also maintain background independence. Otherwise it will not adhere to the experimentally proven and currently understood description of the nature of spacetime (general relativity).

The reason background independence is important specifically to the theory of LQG is that it is the only major theory of quantum gravity that incorporates background independence as a baseline property of the theory. String theory, for example, currently assumes background dependence and hopes to recover background independence at some point in the development of the theory - one of the current shortcomings of string theory as a theory of quantum gravity [5] [10].

II. LOOP QUANTUM GRAVITY'S APPROACH TO MERGING GENERAL RELATIVITY AND QUANTUM FIELD THEORY

According to GR, the gravitational field is one in the same with spacetime itself - gravity is the warping of spacetime. Thus, spacetime itself is a field. Today, nearly all of physics can be described entirely in terms of fields, and by removing background dependence and making spacetime itself a quantum field means that such fields do not exist in or throughout a spacetime, but rather as a mesh of fields on fields on fields [5]. To describe this background independent spacetime-field, one must essentially reconstruct QFT from scratch in a way that does not require a background space to exist. This is the core tenant of LQG.

In the mid 1960s, John Wheeler of Princeton University and Bryce Dewitt (then at North Carolina Chapel Hill) introduced an equation derived from the Hamilton-Jacobi equation of general relativity, now called the Wheeler-DeWitt (WdW) equation [11]. Because of the extremely complicated and abstract mathematics involved (and also because it is way over my head), I will not display the equation but just speak about its conceptual importance to the development of LQG. The WdW equation can be though of as a "wavefunction over geometries" - this means it describes the probability of having one spacetime geometry over another. "Geometry" here is speaking about GR's description of a curved, dynamic spacetime. So, the WdW equation can be thought of as a sort of Schrödinger Equation for the dynamics *of* spacetime and the gravitational field itself - not to be confused with the dynamics of things *in* the field [2, 5, 11].

It turns out that the WdW equation, though a good guide for quantizing space, could not initially produce any nontrivial results [11]. However insight came in the late 1980s when Abhay Ashtekar rewrote GR in terms of a special type of 'connection' field. A connection "enable[s] one to parallel-transport geometrical/physical entities along curves." For example, in electrodynamics, the entity is an electron and the curve is an electric potential. When the object is moved around a closed loop, it will generally be rotated by some amount (see Fig. 2) - and the amount of rotation is a measure of the strength of whatever field the object is moving through (the "curvature of the connection") [12].

With GR written in terms of these new Ashtekar Variables, the WdW equation became more tractable. The old idea of Faraday's "lines of force" (the connections) could be viewed as a quantum excitation of a field, and from this, the WdW equation began to admit a class of exact solutions that seemed to described quantum excitations of the gravitational field: "Ashtekar connection[s] around smooth non-self-intersecting loops." These are the loops of LQG. With mathematical solutions to describe these loops now in hand, LQG could then be defined as "the mathematical description of the quantum gravitational field in terms of these loops" [5, 6, 11].

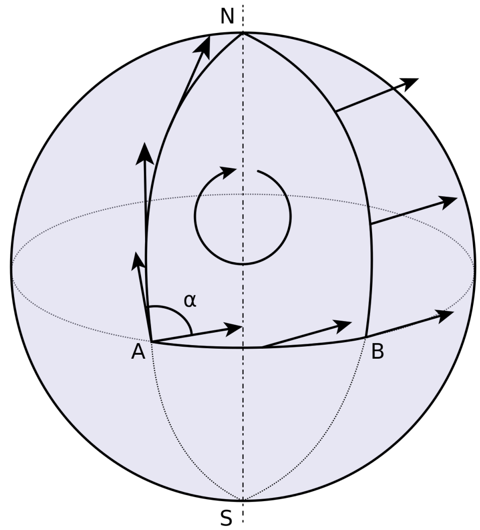


Figure 2: Parallel transport of a vector along a closed loop on a curved surface. As the vector follows the curve and returns to its initial position, it is rotated by some value α. This value of α is a measure of the field strength. Image taken from [13]

III. THE SIGNIFICANCE OF LOOPS

Using Ashtekar Variables to rewrite GR and the WdW equation allowed the floodgates to open for LQG. Once solutions to the WdW equation were found, a solid foundation for a theory of quantum gravity had begun to emerge. Lee Smolin of Yale University and Carlo Rovelli [14] (these two are credited with founding LQG) began to investigate the physical significance of these loops and their implications.

A. Quantizing Space

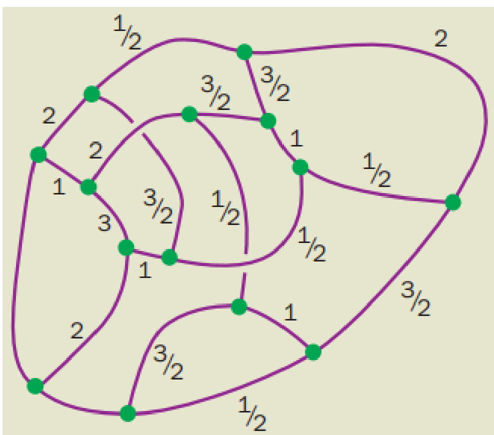
By investigating these loops, Smolin and Rovelli found that these loops were not connections *in* space, but they *are* literally space itself. These loops are quantum excitations of spacetime that weave together like a net to form physical space (see Fig.3). This has the implication that space and time do not exist on any smaller scales than these loops - at scales smaller than this, the concept of space no longer makes sense. Because these loops construct space itself, they are background independent. The 'location' of a loop is determined only by its relative location to the loops its intersects. The whole concept of 'location' at these scales also becomes relative in that "there is no location *of* the net, but only location *on* the net itself; there are no loops on space, only loops on loops." These loops would be on the order of a Planck length $l_p \approx 10^{-35}m$ [2, 5, 6].

Figure 3: Loops are quantum excitations of the gravitational field and thus, spacetime itself (the quantum version of Faraday's lines of force). They weave together to form the fabric, similar to that of chainmail [15]

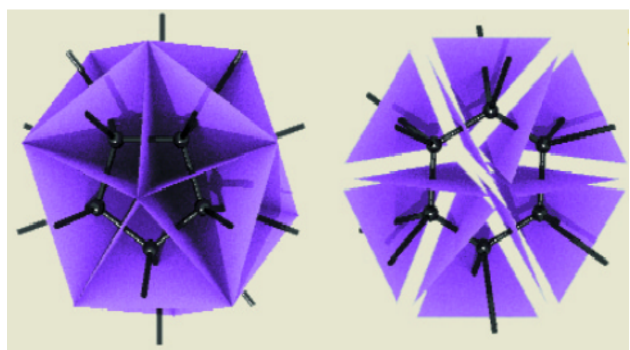
B. Spin Networks

Looking to space beyond one dimension, we must begin to consider the quantization of areas and volumes. In order to quantize something, one must solve an eigenvalue for the operator of the particular physical quantity one is interested. For example, the discrete energy levels of the hydrogen atom are found by finding eigenvalues for a single-proton-single-electron system's hamiltonian (energy operator) through the Schrödinger equation. This is the same process taken to determine the area and volumes elements of loops and their connections.

First, area and volume operators must be created to act upon the mesh of loops (the gravitational field). These operators depend on quantum numbers associated to each loop. More specifically, separating the mesh of loops into a network of 'nodes' (where loops intersect one another) and 'links' (connections between nodes) and attributing each node a quantum number and each link a quantum number. In doing this, Smolin and Rovelli found that these exact same diagrams of links and nodes with quantum numbers had already been developed by Roger Penrose in his own work on quantizing space. Penrose called these combinations of nodes, links, and quantum numbers 'spin networks' because the algebra involved appeared to represent typical spin angular momentum (see Fig. 4). The quantum numbers associated with the links are used to determine the surface area element separating two nodes (see Eq. 7). Similarly, the quantum numbers of the nodes represents the volume of the 'elementary grain of space' in units of the Planck volume (see Eq. 5). These spin networks ultimately represent a quantum state of the local spacetime/the gravitational field. This means that any physical portion of space is in a superposition of these spin network states and, similar to the Schrödinger equation, the Wheeler-Dewitt equation governs the dynamics of the region [5, 7].



(a) A representation of a spin network. Links are the purple lines and nodes are the green dots. Each link has a quantum number associated with it which can be an integer or half integer. Volume quantum numbers can only be integers and are not shown. The link quantum numbers j are used in calculating the area A of the links [5]



(b) A 3-dimensional representation of a spin network. Nodes are the black spheres and links are the black rods. Cells are separated by surfaces in purple. Each surface corresponds to a link. When the surfaces are pulled back, a loop is seen. These are the loops of LQG. These structures build our 3D space.[5]

Figure 4: Two-dimensional and Three-Dimensional visual representations of spin networks

The volume of a loop can be calculated using that node's quantum number λ (I should actually say 'the volume of a node can be calculated...' because volume can only exist *at* the nodes because space does not exist between the links):

$$V = \lambda_i (\frac{G\hbar}{c^3})^{\frac{3}{2}} \tag{5}$$

where λ_i is the quantum number of the i^{th} node [5, 7]. The smallest quantum of volume, the Planck volume can be found when $\lambda_i = 1$:

$$V_{planck} = (\frac{G\hbar}{c^3})^{\frac{3}{2}} = (\sqrt{\frac{G\hbar}{c^3}})^3 = (l_{Planck})^3 \approx 10^{-105} m \tag{6}$$

Area of the links (like the purple surfaces seen in Fig. 4b) are calculated using that link's quantum number j:

$$A = 8\pi\gamma l_p^2 \sqrt{j(j+1)} \tag{7}$$

where $\gamma = \frac{\ln 2}{\sqrt{3}\pi}$ is called the Immirzi parameter, l_p is the Planck length, and j is the quantum number of the link [5, 16, 17]. The smallest non-zero area is given when $j = 1/2$:

$$A_{min} = 8\pi\gamma l_p^2 \sqrt{\frac{1}{2}(\frac{1}{2}+1)} = 4\ln(2) l_p^2 \approx 10^{-70} m^2 \tag{8}$$

Interestingly the minimum area element of a link $A_{min} \neq (A_{Planck} = l_p^2)$, but is still very close $A_{min} \approx l_p^2$.

IV. CONCLUSION

Loop Quantum Gravity is one of the leading attempts to find a theory of quantum gravity alongside String Theory. It adopts a major lesson from general relativity in background independence and using the Wheeler-DeWitt equation in terms of Ashtekar variables it is able to find exact solutions in the form of loops. These loops are found to have a quantized length, area, and volume showing that LQG has made major strides in its attempt to quantize gravity. It has many successes such as quantizing the gravitational field and spacetime, accurately predicting the entropy of a black hole, and removing the singularity of the Big Bang [5]. It's biggest success, though, is that it is able to combine general relativity and quantum mechanics in their current forms without removing many of their core tenants and not adding major assumptions such as extra dimensions or super-symmetry like String Theory [18]. However, LQG is by no means a completed theory and currently, many of its predictions cannot be tested because they extend down to the Planck scale. A more comprehensive review of the theory's shortcomings can be found in [19].

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[2] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity* (Cambridge University Press, 2015).

[3] It also implies that the gravitational field must have an associated force carrying particle, like the photon for the electromagnetic field. This hypothetical particle is called the graviton, but that is not the focus of this paper.

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Appendix A: Response to Reviewer

Here I address the changes made to the original draft of this document as recommended by the reviewers and changes made by the author's own accord.

1. Addressing Peer Reviewers' suggestions and feedback

1. Page 8 - Replaced image for Figure 3 and added citation for where image was obtained
2. Page 9 - Added image citation to Figures 4a and 4b
3. Response to Dr. Ennis's comment "The title page does not count towards the total page length" - I understand. I am writing the document in a preprint format, so it naturally spaces things out for easy review. I do not consider the title page in the document length (See **Changes made under the Author's own accord** item 3).
4. To remove confusion about the meaning of the term "probe" :
   (a) Page 2, between Eq. 1 and Eq. 2 - I changed "say we want to probe some small area with precision L" to "say we want to extract information from an area with precision L"
   (b) Page 2, beginning of paragraph just after Eq. 2 - I changed "probing this small area L" to "Measuring this small area L"
5. Page 5 after Figure 1 to the beginning of section II - Two paragraphs added addressing the importance of background independence to LQG.
6. Top of Page 10 just after Eq. 7 - Removed paragraph about spinfoam. Referee Tye mentioned some confusion about spinfoams, so I have decided to just remove its mention all together. This decision was made because that topic is slightly more advanced and beyond the scope of the paper. Spinfoams are not necessarily required

to obtain a *basic* understanding of Loop Quantum Gravity. So instead of extending the paper to this whole new topic, I have removed the mention of spinfoams all together.

The items above should address all major feedback and concerns given by Referees Dr. Ennis and Sam Tye.

2. Changes made under the Author's own accord

1. Near bottom Page 9, Figure 4 - added caption to Figure 4
2. Very bottom Page 4, Figure 1 - Added caption to figure 1
3. Page 1, Abstract - Added to and refined the abstract to better reflect the paper. In doing this, the structure of the paper fixed itself so that large gap between the title and abstract disappeared.
4. Bottom Page 9 to Section IV - Added some more explanation to volume and area element equations.
5. Top Page 10 - Added Eq. 6 to show that smallest volume element is the Planck volume.
6. Top Page 10 - Rewrote Eq. 7 to match [16, 17]
7. Middle Page 10 - Added Eq. 8 to show minimum area element