Determining the charge to mass ratio of an electron using Zeeman splitting and the polarization of its associated σ lines

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In this paper, we experimentally measure the charge-to-mass ratio of the electron using Zeeman splitting of the 546.1nm emission of mercury and determine the polarization of its associated σ lines. The charge-to-mass ratio is calculated by measuring interference pattern ring diameters when the mercury emission is influenced by a transverse magnetic field and when not subject to a magnetic field. Our result of $\frac{e}{m}$ is accurate to about 1% of the CODATA internationally recommended value. The polarization of the Zeeman split lines are determined with a parallel magnetic field and a quarter wave plate. We observe the produced interference patterns when passed through different angles of a polarizing filter and determine the σ lines to be circularly polarized in different directions.

In 1896, the dutch physicist Pieter Zeeman performed an experiment to determine the influence of a magnetic field on the light signature of sodium. The results of this and a few follow-up experiments found that an applied magnetic field causes spectral line emissions of sodium to separate from individual, fine lines to a 'broadened' form where three distinct lines were visible instead of one [1]. Some 18 years before, in 1878, Hendrik Lorentz proposed a theory that explained the separation of frequencies of light, dispersion, in terms of classical mechanics. He proposed that there exists some kind of small mass, charged particle which was bounded to some larger nucleus such that the charged 'ion' oscillates harmonically within the atom. When Lorentz heard of the results of Zeeman's experiment, he sought to explain the phenomenon with his electromagnetic theory. Together, Zeeman and Lorentz mathematically formulated how the charged particle's oscillation would be effected by a magnetic field. This allowed for a charge-to-mass ratio of the 'ion' to be calculated and provided further evidence that such a charged particle existed. This charged particle became known as the electron [2].

The result of Zeeman and Lorentz's calculation:

$$\frac{T' - T}{T} = \frac{eH}{2k\sqrt{m}} = \frac{e}{m}\frac{HT}{4\pi} \tag{1}$$

Where T is the period of the ion's oscillation; T' is the new period under influence of a magnetic field; H is the intensity of the magnetic field; k is equivalent to Hooke's Law spring constant; e is the charged particle's charge; and m is the charge particle's mass [2].

We calculate the value of $\frac{e}{m}$ by measuring the diameters of concentric interference pattern rings from light emitted by a mercury lamp when a magnetic field is absent and when one is present. Quantum mechanics states that bounded electrons and their behavior can be described by a set of four quantum numbers: Principal (n) - atomic orbital/energy level of electron; Azimuthal(l) - angular momentum of electron; Magnetic(m_l) - energy shift of electron orbital when under the presence of a magnetic field; and Spin(m_s). When a magnetic field is present, it orients the electron's magnetic moment depending on the value of $m_l = 0$ or ± 1 and the polarization of the emitted light (Fig. 1). Selection rules require that the transition of electrons to their new m_l level to be $\Delta M = 0$ or ± 1 This changes the energy of each electron slightly, placing them in different sub-levels of their previous orbital. This slight energy level shift can be observed as a 'splitting' of spectral lines (Fig. 2).

ΔM	Perpendicular to Field	Parallel to Field
+1	Linearly polarized $-\sigma^+$	Circularly polarized $-\sigma^+$
0	Linearly polarized $-\pi$	No light
-1	Linearly polarized $-\sigma^-$	Circularly polarized $-\sigma^-$

FIG. 1. Polarization of emitted light based on magnetic field direction and m_l [3]

By measuring the distance of the line splitting, one can calculate $\frac{e}{m}$:

$$\frac{e}{m} = (\frac{2\pi c}{dB})(\frac{1}{M_2 g_2 - M_1 g_1})(\frac{D_k'^2 - D_k^2}{D_{k-1}^2 - D_k^2}) \tag{2}$$

Where c is the speed of light, d is the separation of the interferometer mirrors, B is the strength of the applied magnetic field, M_n is the magnetic quantum number and g_n is the Landé g factor (in our case $M_2 g_2 - M_1 g_1 = 2 - \frac{3}{2} = \frac{1}{2}$), and the distances between lines

$$D_k'^2 - D_k^2 = ((K_{outer}^2 - K_{mid}^2) + (K - 1_{outer}^2 - K - 1_{mid}^2) + (K - 2_{outer}^2 - K - 2_{mid}^2))/3 \tag{3}$$

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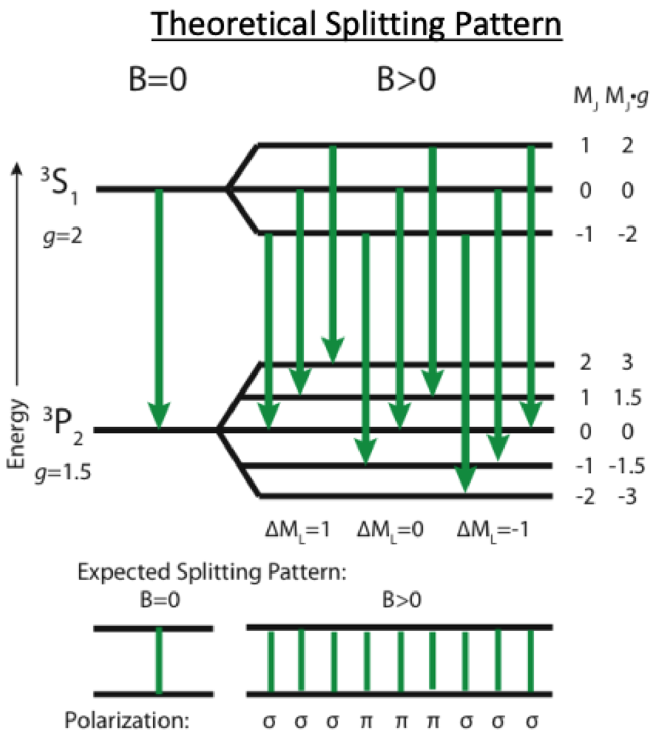


FIG. 2. Model of Zeeman splitting of mercury 546.1nm from one lines into nine lines [4]

and

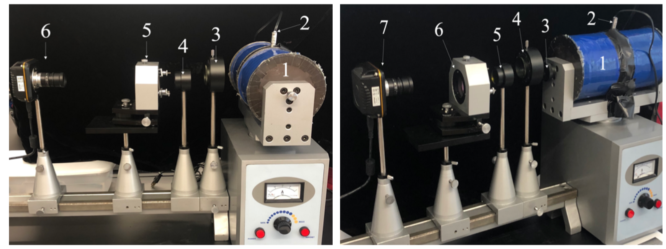
$$D^2_{k-1} - D^2_k = ((K - 1^2_{ring} - K^2_{ring}) + (K - 2^2_{ring} - K - 2^2_{ring}))/2 \quad (4)$$

where $K - n_{ring}$ is the interference pattern rings without an applied magnetic field, $n = 0, 1, 2$ is the most central ring, the next ring laterally out, and the next ring after that, respectively. And $K - n_{inner,mid,outer}$ are the Zeeman split lines of the $K - n_{ring}$ with $inner, mid, outer$ indicating the most central split line, middle line, and outermost split line, respectively.

To calculate $\frac{e}{m}$ we measured the Zeeman splitting of the mercury lamp's emission using a transverse magnetic field in **Setup A** and we determined the polarization of the light with a parallel magnetic field in **Setup B**.

The magnet is a circular helmholtz coil which can produce a uniform magnetic field. The maximum current that can be run through the coil is $1.40 \pm 0.05A$ as indicated by the manufacturer. This produces a maximum magnetic field of $1.074 \pm 0.054T$ as measured with a Hall-effect probe. The lens/polarizing filter will filter certain linearly polarized light depending on how it is rotated. The interference filter only transmits light of wavelength $= 546.1nm$ and reflects all other wavelengths. We use a Fabry-Pérot interferometer that uses two partially reflective quartz plates separated by $2.00 \pm 0.01mm$ to produce an interference pattern - concentric rings of light. This interference pattern is observed live with a CCD camera

Setup A | **Setup B**



Setup A:

1. Magnet
2. Mercury Lamp
3. Lens/Polarizing Filter
4. Interference Filter
5. Interferometer (Fabry-Pérot)
6. CCD Camera connected to computer

Setup B:

1. Magnet
2. Mercury Lamp
3. $\lambda/4$ Polarizer
4. Lens/Polarizing Filter
5. Interference Filter
6. Interferometer (Fabry-Pérot)
7. CCD Camera connected to computer

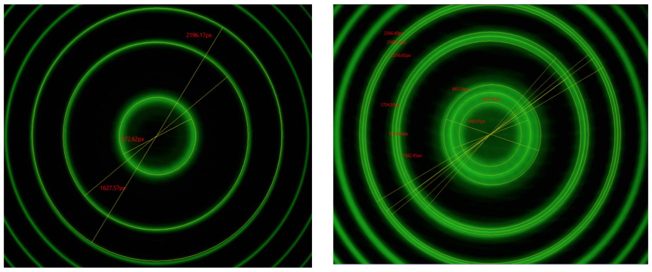
FIG. 3. Experimental Setups used to measure Zeeman splitting (**A**) and polarization of the emitted light (**B**)

and is displayed on a computer monitor; the camera's gain and exposure can be adjusted to better view the interference pattern.

For **Setup A**, the mercury lamp is centered in the space between the helmholtz coil magnet so that the light emitted will be uniformly influenced by the magnetic field. Next, all the components of the experiment are aligned such that the emitted light will be focused on the camera lens. The lamp and the camera are turned on and the setup is fine tuned to display the interference pattern. Once we believed the pattern shown on the screen was good enough to analyze (clear interference pattern, sharp rings, appropriate exposure), we took a snapshot of the pattern with the camera. This image could then be analyzed by measuring the diameter of each concentric ring in pixels. When no magnetic field is applied (Fig. 4a), an interference pattern of concentric rings is seen. When a magnetic field is applied (Fig. 4b), each individual K, K-1, K-2, etc. ring splits into 3 distinct lines. In reality, the individual lines actually split into 9 lines (3 π and 6 σ), however the 6 σ lines are much weaker and difficult to observe (Fig. 5).

The measurements of the ring diameters were done an additional 6 times using the same interference patterns as displayed in Fig. 4. Using these measurements, we found the standard deviation of each ring measurement to be ± 8.8 pixels.

For **Setup B**, we followed the same process of operating the lamp, magnet and lenses/filters to produce interference patterns. **Setup B**, however, differs from **setup A** in that the magnet has been rotated 90° so the magnetic field is now parallel to the lamp's emitted light and it has an additional $\lambda/4$ filter at position 3. Light is a combination of oscillating electric and magnetic fields. Each of these components is a vector and thus can be represented as a combination of two other vectors. The $\lambda/4$ filter essentially shifts one of these vector components by a quarter of a wavelength which alters the resultant



(a) Interference pattern with no applied magnetic field

(b) Interference pattern applied transverse magnetic field = 1.074±0.054T

(a)	K			K-1			K-2		
	672.6			1627.5			2196.1		
(b)	K_{inner}	K_{mid}	K_{outer}	K-1_{inner}	K-1_{mid}	K-1_{outer}	K-2_{inner}	K-2_{mid}	K-2_{outer}
	540	697.1	843.5	1582.4	1644.8	1704.9	2216.6	2169.1	2266.4

FIG. 4. Interference pattern of Mercury without **(a)** and with **(b)** an applied transverse magnetic field. Measured ring diameters presented in accompanying table - all values in pixels. Standard deviation of all diameter measurements is ±8.8 px

electric and magnetic field vectors. This has the effect of converting linearly polarized light into circularly polarized light and vice versa. By filtering the resulting light with the lens/polarizing filter at position 4, and viewing the resulting interference pattern, we can determine the orientation of the σ lines (right-handed or left-handed).

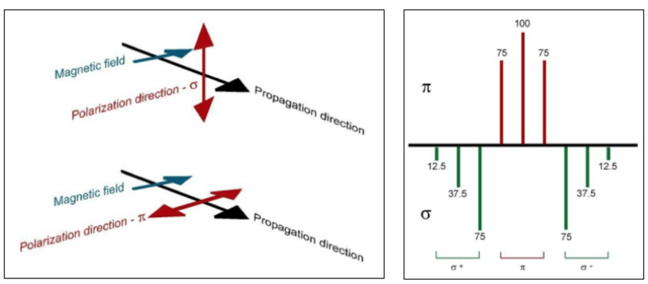


FIG. 5. Orientation of σ and π polarized light with a transverse magnetic field like in **setup A**. The right image shows the 9 lines produced from Zeeman splitting consist of three strong π-polarized lines and six, weaker σ-polarized lines.

We captured images of the interference pattern with the lens/polarizing filter rotated clockwise at $\theta = 0°$, 45°, 90°, and 135° as viewed from the camera (Fig. 6).

We calculate the charge-to-mass ratio of the electron using the Zeeman split ring diameters measured in Fig. 4 and equation 2 (and equations 3 & 4) to be:

$$\frac{e}{m} = (-1.74 \pm 0.12) * 10^{11} c/kg \qquad (5)$$

This is approximately a 1% difference from the internationally recommended NIST/CODATA value of $-1.75(8) * 10^{11} c/kg$ [5] and is within the error of our experiment. We obtained our error of $\pm 1.2 * 10^{10} c/kg$ by taking our calculated standard deviation of ring diameter

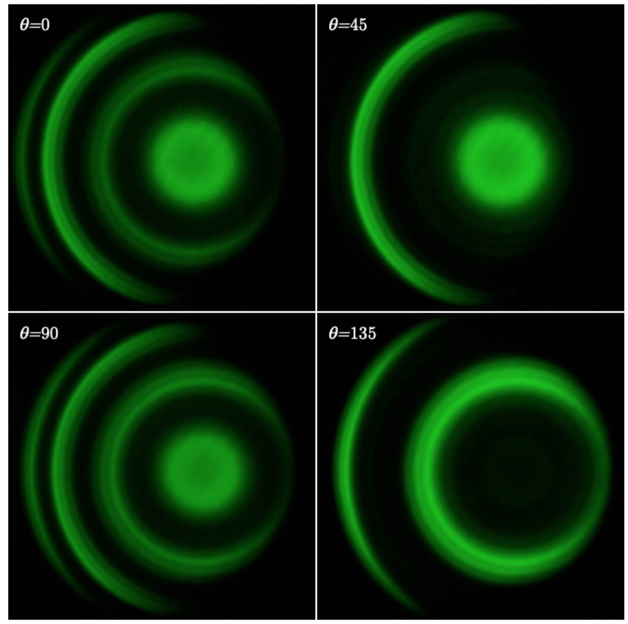


FIG. 6. Interference patterns from **setup B** with the lens/polarizing filter located at position 4 rotated at $\theta = 0°$, 45°, 90°, and 135°.

measurements (±8.8 px) and applying this to each value used in equations 3 & 4 to get upper and lower bounds for the value produced by equation 2.

We are also able to determine the polarization of the emitted σ lines. We know that the three middle Zeeman split lines are linearly polarized π lines. When we rotate the magnet 90° from **setup A** to **setup B**, such that the magnetic field is now parallel to the propagation direction, the oscillations of the π lines' electric field becomes aligned with the propagation direction. This means the π lines are not observed by the camera and so are not present in any of the images in Fig 6. This lack of π lines is seen at $\theta = 0°$ and 90° as large gaps between the rings. The rings that are visible are the six σ^+ and σ^- lines. However, if we rotate the polarizing filter 45°, the second and fourth rings disappear leaving only the first and third rings visible. At 135°, the first and third rings disappear and the second and fourth rings are the only ones visible. This implies that the σ^+ lines are circularly polarized in one direction while the σ^- lines are circularly polarized in the opposite direction (opposite handedness).

Their handedness can be determined by the angle of the polarizing filter and which σ line it corresponds to. At 45° the most central lines remain, implying these are the σ^- lines (Fig. 5). If we then take the reference frame of the light (moving through the filter towards the camera) the grating of the polarizing filter is angled such that it points to the upper left and lower right. Thus, the linearly polarized light that is produced from the quarter wave plate is angled 45° to the left from the vertical. Thus, the σ^- lines are left-handed circularly polarized. Therefore, the σ^+ lines are right-handed circularly polarized.

In summary, we calculated the charge-to-mass ratio of the electron to about 1% of the internationally recognized value by measuring the interference pattern ring diameters of the 546.1nm Zeeman split lines of mercury. We also determined the polarization of the accompanying σ^+ to be right-hand circularly polarized while the σ^+ was left-hand circularly polarized.

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